RESEARCH ARTICLE

Fuzzy Supra α - Baire Spaces

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ABSTRACT: In this paper the concepts of fuzzy supra α-Baire spaces are introduced and characterizations of fuzzy supra α- Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS: Fuzzy supra α- open set, Fuzzy supra α- nowhere dense set, Fuzzy supra α- first category, Fuzzy supra α- second category, Fuzzy supra α-Baire spaces.

https://doi.org/10.29294/IJASE.5.1.2018.857-861

1. INTRODUCTION

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by Zadeh [1]. The theory of fuzzy topological spaces was introduced and developed by Chang [2]. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy has been found to be very useful in solving many practical problems. El.Naschie [3] showed that the notion of fuzzy topology might be relevant to Quantum Particle Physics and Quantum Gravity in connection with string theory and $\in^{\infty}$ theory. Similarly, Tang [4] used a slightly changed version of Chang' s fuzzy topological spaces to model spatial objects for GIS data bases and Structured Query Language (SQL) for GIS. In this paper, we introduce the concepts of fuzzy supra α-Baire spaces. Also we discuss several characterizations of fuzzy supra α-Baire spaces. Several examples are given to illustrate the concepts introduced in this paper.

2. PRELIMINARIES

Now we introduce some Basic notions and results that are used in the sequel. In this work we shall mean a non-empty set X together with a fuzzy supra topology T* and denote it by (X,T*) .

Definition 2.1[1]: A Fuzzy set in X is a map f: X→[0,1]= I. The family of fuzzy sets of X is denoted by I^X. Following are some basic operations in fuzzy sets in X. For the fuzzy sets f and g in X,

1) $f = g$ if and only if $f(x) = g(x)$ for all $x\in X$
2) $f \le g$ if and only if $f(x) \le g(x)$ for all $x\in X$
3) $(f\vee g)(x)= \max\{f(x), g(x)\}$ for all $x\in X$
4) $(f\wedge g)(x)= \min\{f(x), g(x)\}$ for all $x\in X$
5) $f^c(x) = 1-f(x)$ for all $x\in X$ here f^c denotes the complement of f.
6) For a family $\{f_\lambda/\lambda\in\Lambda\}$ of fuzzy sets defined on a set X
$$(\vee_{\lambda\in\Lambda}f_\lambda)(x) = \vee_{\lambda\in\Lambda}(f_\lambda(x))$$
$$(\wedge_{\lambda\in\Lambda}f_\lambda)(x) = \wedge_{\lambda\in\Lambda}(f_\lambda(x))$$
7) For any $\alpha \varepsilon I$, the constant fuzzy set α in X is a fuzzy set in X defined by $\alpha(x)=\alpha$ for all $x\in X$.
8) 0 denotes null fuzzy set in X and 1 denotes universal fuzzy set in X.

Definition 2.2[2]: A fuzzy topological space is a pair (X,δ) where X is a nonempty set and δ is a family of fuzzy set on X satisfying the following properties:

1) The constant functions 0 and 1 belong to δ.
2) $f,g \in\delta$ implies $f\wedge g \in\delta$.
3) $f_\lambda\in\delta$ for each $\lambda\in\Lambda$ implies$(\vee_{\lambda\in\vee} f_\lambda)\in\delta$.

Then δ is called a fuzzy topology on X. Every member of δ is called fuzzy open set. The complement of a fuzzy open set is called fuzzy closed set.

Definition 2.3[2]: The closure and interior of a fuzzy set $f\in I^X$ are defined respectively as

$cl(f)= \wedge\{g/g$ is a fuzzy closed set in X and $f\le g\}$
$int(f)= \vee\{g/g$ is a fuzzy open set in X and $g\le f\}$

Clearly cl(f) is the smallest fuzzy closed set containing f and int(f) is the largest fuzzy open set contained in f.

Definition 2.4[5]: A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied:

1) 0 and 1 belongs to δ^*.
2) $f_\lambda\in\delta^*$ for each $\lambda\in\wedge$ implies $(\vee_{\lambda\in\wedge}f_\lambda)\in\delta^*$.

The pair (X,δ^*) is called a fuzzy supra topological space. The elements of δ^* are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition 2.5[5]: let (X,δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior of f defined respectively as

$cl^*(f)= \wedge\{g/g$ is a fuzzy supra closed set in X and $f \le g\}$
$int^*(f)=\vee\{g/g$ is a fuzzy supra open set in X and $g \le f\}$

Definition 2.6[5]: let (X,δ) be a fuzzy topological space and δ^* be a fuzzy supra topology on X. We call δ^* a fuzzy supra topology associated with δ if $\delta \le \delta^*$

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Received: 01.05.2018 **Accepted:** 18.07.2018 **Published on**: 24.08.2018

Remark 2.7[5]:

1) The fuzzy supra closure of a fuzzy set f in a fuzzy supra topological space is the smallest fuzzy supra closed set containing f.
2) The fuzzy supra interior of a fuzzy set f in a fuzzy supra topological space is the largest fuzzy supra open set contained in f.
3) If (X,δ^*) is an associated fuzzy supra topological space with the fuzzy topological space (X,δ) and f is any fuzzy set in X, then
   $int(f) \le int^*(f) \le f \le cl^*(f) \le cl(f)$

Theorem 2.8[5]: For any fuzzy sets f and g in a fuzzy supra topological space (X,δ^*),

1) f is a fuzzy supra closed set if and only if $cl^*(f)=f$.
2) f is a fuzzy supra open set if and only if $int^*(f)=(f)$
3) $f \le g$ implies $int^*(f) \le int^*(g)$ and $cl^*(f) \le cl^*(f) \le g$
4) $cl^*(cl^*(f)) = cl^*(f)$ and $int^*(int^*(f)) = int^*(f)$.
5) $cl^*(f \vee g) \ge cl^*(f) \vee cl^*(g)$
6) $cl^*(f \wedge g) \le cl^*(f) \wedge cl^*(g)$
7) $int^*(f \vee g) \ge int^*(f) \vee int^*(g)$
8) $int^*(f \wedge g) \le int^*(f) \wedge int^*(g)$
9) $cl^*(f^c) = [int^*(f)]^c$
10) $int^*(f^c) = [cl^*(f)]^c$

Definition 2.9[5]: Let (X,δ^*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra semi open set if, $f \le cl^*(int^*(f))$. The complement of a fuzzy supra semi open set is called a fuzzy supra semi closed set.

Definition 2.10[6]: Let (X,T)be any fuzzy topological space and λ be any fuzzy set in (X,T). We define $cl(\lambda) = \wedge \{\mu/\lambda \le \mu, 1-\mu \in T\}$ and $int(\lambda) = \vee\{\mu/\mu \le \lambda, \mu \in T\}$. For any fuzzy set in a fuzzy topological space (X,T) it is easy to see that $1-cl(\lambda)=int(1-\lambda)$ and $1-int(\lambda)=cl(1-\lambda)$.

Definition 2.11[7]: Let (X, T*) be a fuzzy supra topological space. A fuzzy set f is called fuzzy supra α - open set, if $f \le int^*[cl^*(int^*(f))]$ and a fuzzy set f is called fuzzy supra α - closed set if $cl^*[int^*(cl^*(f))] \le f$.

Definition 2.12[8]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy α - dense if there exists no fuzzy α - closed set μ in (X,T) such that $\lambda<\mu<1$.

Definition 2.13[8]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non - zero fuzzy open set μ in (X,T) such that $\mu<cl(\lambda)$. That is, $int\, cl(\lambda)=0$.

Definition 2.14[9]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where λ_i's are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.15[10]: A fuzzy topological space (X,T) is called fuzzy first category space if $1 = \vee_{i=i}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy nowhere dense sets in(X,T). A topological space which is not of fuzzy first category is said to be of fuzzy second category space.

Definition 2.16[9]: Let λ be a fuzzy first category set in a fuzzy topological space (X, T). Then $1-\lambda$ is called a fuzzy residual set in (X, T).

Definition 2.17 [9]: Let (X,T) be a fuzzy topological space.Then (X,T) is called a fuzzy baire space if $int(\vee_{i=i}^{\infty}(\lambda_i)) = 0$, where λ_i's are fuzzy nowhere dense sets in (X,T).

3. FUZZY SUPRA α- NOWHERE DENSE SETS

Definition 3.1: Let (X, T*) be a fuzzy supra topological space. A fuzzy set λ in (X, T*) is called a fuzzy supra α - nowhere dense set if there exixts no non-zero fuzzy supra α - open set μ in (X,T*) Such that $\mu<\alpha\text{-}cl^*(\lambda)$. That is, $\alpha\text{-}int^*\,\alpha\text{-}cl^*(\lambda)=0$.

Definition 3.2: A fuzzy set λ in a fuzzy supra topological space (X,T*) is called fuzzy supra α- first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy supra α-nowhere dense set in (X,T*). Any other fuzzy set in (X, T*) is said to be of fuzzy supra α-second category.

Definition 3.3: Let λ be a fuzzy supra α- first category set in a fuzzy supra topological space (X, T*). Then $1-\lambda$ is called a fuzzy supra α- residual set in (X, T*).

Example 3.4: Let X= {a, b, c}. The fuzzy supra set β; γ and δ are defined on X as follows:

$\beta: X \to [0,1]$ define as $\beta(a)=0,9$; $\beta(b)=0.7$; $\beta(c)=0.7$,
$\gamma: X \to [0,1]$ define as $\gamma(a)=0.8$; $\gamma(b)=0.7$; $\gamma(c)=0.6$,
$\delta: X \to [0,1]$ define as $\delta(a)=0.9$; $\delta(b)=0.9$; $\delta(c)=0.8$.

Then $T= \{0, \beta, \gamma, \delta, \beta\vee\delta, \beta\wedge\delta, 1\}$ is clearly a fuzzy supra topology on X. Now consider the following fuzzy sets defined on X as follows:

$\lambda: X \to [0,1]$ define as $\lambda(a)=0.9$; $\lambda(b)=0.9$; $\lambda(c)=0.8$,
$\mu: X \to [0,1]$ define as $\mu(a)=0.8$; $\mu(b)=0.7$; $\mu(c)=0.7$,
$\eta: X \to [0,1]$ define as $\eta(a)=0.8$; $\eta(b)=0.8$; $\eta(c)=0.7$.

Then the fuzzy supra α-open sets in (X,T*) are β, γ, δ, $\beta\vee\delta$, $\beta\wedge\delta$, λ, μ, η.
Thus $\alpha\text{-}int^*\alpha\text{-}cl^*(1-\beta)=0$, $\alpha\text{-}int^*\,\alpha\text{-}cl^*(1-\gamma)=0$, $\alpha\text{-}int^*\,\alpha\text{-}cl^*(1-\delta)=0$, $\alpha\text{-}int^*\,\alpha\text{-}cl^*[1-(\beta\vee\delta)]=0$, $\alpha\text{-}int^*\,\alpha\text{-}cl^*[1-\beta\wedge\delta)]=0$. So $1-\beta$, $1-\delta$, $1-(\beta\vee\delta)$, $1-(\beta\wedge\delta)$ are fuzzy supra α-nowhere dense sets in (X,T*).
But λ, μ, η are not fuzzy supra α- nowhere dense sets, since $\alpha\text{-}int^*\alpha\text{-}cl^*(\lambda)=1 \neq 0$, $\alpha\text{-}int^*\alpha\text{-}cl^*(\mu)=1 \neq 0$, $\alpha\text{-}int^*\alpha\text{-}cl^*(\eta)=1\neq 0$.
On one hand,
$(1-\beta) \vee (1-\gamma) \vee (1-\delta) \vee [1-(\beta\vee\delta)] \vee [1-(\beta\wedge\delta)]=1-\delta$ is fuzzy supra α- first category set and δ is a fuzzy supra α-residual set in (X,T*).

Proposition 3.5: If λ is a fuzzy supra α- closed set in (X,T*) with $\alpha-int^*(\lambda)=0$ then λ is a fuzzy supra α- nowhere dense set in (X,T*).

proof: Let λ be a fuzzy supra α-closed set in (X,T*). Then $\alpha\text{-}cl^*(\lambda)=\lambda$. Now $\alpha\text{-}int^*\,\alpha\text{-}cl^*(\lambda)=\alpha\text{-}int^*(\lambda)=0$.Thus λ is a fuzzy supra α- nowhere dense set in (X,T*).

Proposition 3.6: If λ is a fuzzy supra α- nowhere dense set in (X,T*),then $\alpha\text{-}int^*(\lambda)=0$.

Proof: Let λ be a fuzzy supra α-nowhere dense set in (X,T*). Now $\lambda \le \alpha$-cl*(λ) implies that α-int*(λ) $\le$ α-int* α-cl*(λ)=0 Then α-int*(λ)=0.

Remark 3.7: The converse of the above proposition need not be true, for consider the following example.

Example 3.8: Let X = {a.b}. The fuzzy supra sets β, γ and δ are defined on X as follows:

β:X→[0,1] defined as β(a)=0.8; β(b)=0.6,
γ:X→[0,1] defined as γ (a)=0.9; γ (b)=0.8,
δ:X→[0,1] defined as δ (a)=0.9; δ (b)=0.6.

Then T={0, β,γ ,δ,1} is a fuzzy supra topology on X. Now consider the following fuzzy sets defined on x as follows:
λ: X→[0,1] defined as λ (a)=0.9, λ (b)=0.5.Thus the fuzzy supra α-open sets in (X,T*) are β ,γ ,δ. Now α- int*(λ) =0 where as α-int* α-cl*(λ) =1 $\neq$0. Hence λ is not a fuzzy supra α-nowhere dense set in (X, T*).

Remark 3.9: The complement of a fuzzy supra α- nowhere dense set need not be a fuzzy supra α-nowhere dense set. For consider the following example.

Example 3.10: Let X={a,b} and the fuzzy supra set β, γ be defined on X as follows:

β:X→[0,1] defined as β(a)=0.9; β(b)=0.8,
γ:X→[0,1] defined as γ (a)=0.9; γ (b)=0.9.

Then T={0, β, γ,1} is a fuzzy supra topology on X. The fuzzy supra α-open sets in (X,T*) are β, γ. Now 1–γ is a fuzzy supra α-nowhere dense set in (X,T*) where as γ is not a fuzzy supra α-nowhere dense set, since α-int* α-cl*(μ)= α-int*(1)=1 $\neq$0.

Proposition 3.11: If λ and μ are fuzzy supra α- nowhere dense sets in (X, T*), then ($\lambda \wedge \mu$) is a fuzzy supra α- nowhere dense set in (X, T*).

Proof: let λ and μ are fuzzy supra α- nowhere dense sets in (X, T*). Then α-int* α-cl*($\boldsymbol{\lambda}\wedge$ μ) $\le$ α-int* α-cl*(λ) $\wedge$ α-int* α-cl*(μ)=0 $\wedge$0=0. Since λ and μ are fuzzy supra α- nowhere dense sets in (X,T*). Thus $\lambda \wedge \mu$ is fuzzy supra α- nowhere dense sets in (X, T*).

Proposition 3.12: let λ and μ are fuzzy supra α- nowhere dense sets,then ($\lambda \vee \mu$) need not be a fuzzy supra α- nowhere dense set in (X,T*). For consider the following example.

Example 3.13 Let X={a,b,c}. The fuzzy supra sets β, γ and δ are defined on X as follows:

β:X → [0,1] defined as β(a)=0.7; β(b)=0.4; β(c)=1,
γ:X → [0,1] defined as γ (a)=1; γ (b)=0.2; γ (c)=0.7,
δ: X → [0,1] defined as δ(a)=0.3; δ(b)=0.1; δ(c)=0.2.

Then T= {0, β, γ, δ, $\beta\vee\gamma$, $\beta\wedge\gamma$, 1} is clearly fuzzy supra topology on X. Thus the fuzzy supra α- open sets in (X, T*) are β, γ, δ, $\beta\vee\gamma$, $\beta\wedge\gamma$. Now 1–β and 1–γ are fuzzy supra α-nowhere dense sets in (X, T*). But [(1–β) $\vee$ (1– γ)] = 1–($\beta\wedge\gamma$) is not a fuzzy supra α-nowhere dense sets in (X, T*). Since α-int* α-cl*[1–($\beta\wedge\gamma$)] $\neq$0, (1–β)$\vee$ (1– γ) is not a fuzzy supra α-nowhere dense sets in (X,T*).

Proposition 3.14: If $\boldsymbol{\lambda}$ is a fuzzy supra α-dense, fuzzy supra α- open set in (X,T*). Such that $\mu \le (1 - \boldsymbol{\lambda})$ then μ is a fuzzy supra α- nowhere dense set in (X,T*).

Proof: Let λ be a fuzzy supra α- open in (X,T*). Such that α – cl*(λ) =1.
Now $\mu\le(1 - \lambda)$ implies that α-cl*(μ)$\le$ α-cl*(1 – λ)=1 – λ [since (1 – λ) is fuzzy supra α - closed in (X,T*)].
Then we have α-int* α-cl*(μ) $\le$ α- int*(1 – λ)=1 – α -cl*(λ)=1–1 = 0.
Thus α-int*α-cl*(μ) =0. So μ is a fuzzy supra α-nowhere dense set in (X,T*).

Proposition 3.15: If a non - zero fuzzy supra set λ in (X,T*) is a fuzzy supra α-nowhere dense set then λ is fuzzy supra semi-closed in (X,T*).

Proof: let λ be fuzzy supra α-nowhere dense set in (X, T*). Then α-int* α-cl*(λ) =0. Thus α-int* α-cl* (λ) $\le \lambda$. so λ is a fuzzy supra semi - closed set in (X, T).

Remark 3.16: The converse of the above proposition need not be true. For consider the following example.

Example 3.17: let X={a,b,c} The fuzzy supra sets β and γ are defined on X as follows:

β:X → [0,1] defined as β(a)=0.4; β(b)=0.7; β(c)=0.6,
γ:X → [0,1] defined as γ (a)=0.6; γ (b)=0.5; γ (c)=0.8.

Then T= {0, β, γ, $\beta \vee\gamma$, $\beta \wedge\gamma$, 1} is a fuzzy supra topology on X. The fuzzy supra α- open sets in (X, T*) are β, γ, $\beta \vee\gamma$, $\beta \wedge\gamma$. Now $\beta \wedge\gamma$ is a fuzzy semi- closed set where as $\beta\wedge\gamma$ is not a fuzzy supra α-nowhere dense set, since α- int*α-cl*($\beta\wedge\gamma$) = β $\neq$0.

Proposition 3.18: let λ is a fuzzy supra α- closed set in (X,T*),Then λ is a fuzzy supra
α-Nowhere dense set in (X,T*), if and only if α-int*(λ)=0.

Proof: let λ be a non - zero fuzzy supra α-closed set in (X, T*) with α-int*(λ)=0.Then by proposition 3.5, λ is a fuzzy supra α-nowhere denseset in (X,T*). Conversely let λ be a fuzzy supra α-nowhere dense set in (X, T*). Then α-int* α-cl*(λ)=0. Which implies that α –int* (λ)=0. [Since λ is a fuzzy supra α-closed, α-cl*(λ) = λ].

Proposition 3.19: If $\boldsymbol{\lambda}$ is a fuzzy supra α-nowhere dense set in (X, T*), Then (1 –λ) is a fuzzy supra α- dense set in (X, T*).

Proof: Let λ be a fuzzy supra α- dense set in (X, T*). Then, by Proposition 3.6, we have
α-int*(λ)=0. Now α-cl*(1 – λ) =1 – α-int*(λ) =1–0=1.Thus 1 –λ is a fuzzy supra α- dense set in (X, T*).

Proposition 3.20: If λ is a fuzzy supra α- dense and fuzzy supra α-open set in (X, T*). Then 1 – λ is fuzzy supra α-nowhere dense set in (X, T*).

Proof: Let λ be a fuzzy supra α- open set in (X, T*) such that α-cl*(λ) =1. Now int* α-cl*
(1 – λ)=1 – α-cl* α-int*(λ) =1 – α-cl*(λ) =1 –1= 0. Then 1 –λ is a fuzzy supra α-nowhere dense set in (X, T*).

Proposition 3.21: If $\boldsymbol{\lambda}$ is a fuzzy supra α-nowhere dense set in (X,T*). Then α-cl*(λ) is a fuzzy supra α- nowhere dense set in (X,T*).

Proof: Let α-cl*(λ)=μ. Now α-ınt* α-cl*(μ)= α-int* α-cl*(cl* (λ)) = α-int* α-cl*(λ)=0
Then μ= α-cl*(λ) is a fuzzy supra α-nowhere dense set in (X, T*).

Proposition 3.22: If λ be a fuzzy supra α-nowhere dense set in (X, T*), then 1 –α-cl*(λ) is a fuzzy supra α -dense set in (X, T*).

Proof: Let λ be a fuzzy supra α-nowhere dense set in (X,T*), by proposition 3.21, we have α-cl*(λ) is a fuzzy supra α-nowhere dense set in (X,T*) by proposition 3.19, we have 1 – α-cl*(λ) is a fuzzy supra α- dense set in (X,T*)

Proposition 3.23: Let λ be a fuzzy supra α-dense set in a fuzzy supra topological space (X,T*). If μ is any fuzzy supra set in (X,T*), then μ is fuzzy supra α-nowhere dense set in (X.T*) if and only if (λ ∧ μ) is a fuzzy supra α- nowhere dense set in (X,T*).

Proof:Let μ be a fuzzy supra α-nowhere dense set in (X.T*), Then α-int*α-cl*(λ ∧ μ) = α-int* [α-cl*(λ) ∧ α-cl*(μ)]= α-int* [1 ∧α-cl*(μ)]= α-int* α-cl*(μ)= 0.
Then (λ ∧ μ) is a fuzzy supra α- nowhere dense set in (X.T*). Conversely, let λ ∧μ be a fuzzy supra α- nowhere dense set in (X.T*). Then α-int*α-cl*(λ ∧ μ) =0. Implies that α-int*[α-cl*(λ) ∧ α-cl*(μ)]=0.
Thus α-int*[1 ∧ α-cl*(μ)] =0 so α-int* α-cl*(μ) =0 which means that μ is a fuzzy supra α-nowhere dense set in (X.T*).

Proposition 3.24: If λ ≤ μ and μ is a fuzzy supra α- nowhere dense set in a fuzzy supra topologoical space (X.T*), then λ is also a fuzzy supra α-nowhere dense set in (X.T*).

Proof: Now λ ≤ μ implies that α-int* α-cl*(λ) ≤ α-int* α-cl*(μ). Now μ is a fuzzy supra α-nowhere dense set implies that α-int* α-cl*(μ) = 0. Then α-int* α-cl*(λ) = 0. Thus λ is a fuzzy supra α-nowhere dense set in (X, T*) .

Proposition 3.25: If λ ≤ μ and μ is a fuzzy supra α- first category set in a fuzzy supra topological space (X, T*), Then λ is also a fuzzy supra α - first category set in (X,T*).

Proof: Let μ_i be a fuzzy supra α-first category set in (X, T*). Then μ= $\vee_{i=1}^{\infty}$(μ_i) where (μ_i)' s are fuzzy supra α-nowhere dense set in (X, T*). Now λ ∧ μ gives λ ∧ μ =λ implies that λ∧μ≤ μ.
Then by proposition 3.24, λ ∧ μ is a fuzzy supra α-nowhere dense set in (X, T*). That is λ=$\vee_{i=1}^{\infty}$(λ∧μ_i) and (λ ∧ μ_i) ' s are fuzzy supra α-nowhere dense set in (X,T*) gives λ is also a fuzzy supra α- first category set in (X,T*).

Proposition 3.26: If λ ≤ μ and λ is a fuzzy supra α-residual set in a fuzzy supra topological space(X, T*), then μ is also a fuzzy supra α-residual set in (X, T*).

Proof: Let λ be a fuzzy supra α - residual set in (X, T*). Then 1 – λ is a fuzzy supra α- first category set in(X, T*), Let η=1 – λ is a fuzzy supra α- first category set in(X, T*). Now λ ≤ μ implies that 1 –η≤ μ Then η ≥ 1–μ since η is a fuzzy supra α- first category set in(X,T*).
By proposition 3.25, 1 – μ is a fuzzy supra α- first category set in(X,T*), and hence μ is a fuzzy supra α- residual set in (X,T*).

4. FUZZY SUPRA α- BAIRE SPACE

DEFINITION 4.1: Let (X,T*) be a fuzzy supra topological space. Then (X,T*) is called a fuzzy supra α-Baire space if α- int* ($\vee_{i=1}^{\infty}$(λ_i))=0 where (λ_i)'s are fuzzy supra α- nowhere dense sets in (X,T*).

Example 4.2: Let X={ a, b, c}. the fuzzy supra sets β, γ and δ are defined on X as follows:

β:X → [0,1] defined as β(a)=0.9; β(b)=0.8; β(c)=0.8,
γ:X → [0,1] defined as γ (a)=0.8; γ (b)=0.8; γ (c)=0.7,
δ: X → [0,1] defined as δ (a)=0.9; δ (b)=0.9; δ(c)=0.8.

Then T={ 0,β, γ ,δ, 1} is clearly a fuzzy supra topology on X. Thus the fuzzy supra α- open sets in (X,T*) are β, γ ,δ. So α-int*α-cl*(1– β)=0, α-int*α-cl*(1– γ)=0, α-int*α-cl*(1– δ)=0. so 1– β, 1– γ ,1– δ are fuzzy supra α-nowhere dense sets in (X,T*) implies that α-int*[(1–β) ∨ (1– γ) ∨ (1– δ)] = 0. Hence (X,T*) is a fuzzy supra α-Baire space.

Example 4.3: Let X={ a, b, c}. The fuzzy supra sets β, γ and δ are defined on X as follows:

β:X → [0,1] defined as β(a)=0.7; β(b)=0.4; β(c)=1,
:X → [0,1] defined as γ (a)=1; γ (b)=0.2; γ (c)=0.7,
δ: X → [0,1] defined as δ(a)=0.3; δ(b)=0.1;δ(c)=0.2.

Then T={ 0,β, γ ,δ, β ∨γ, β ∧γ , 1} is clearly a fuzzy supra topology on X. Thus the fuzzy supra α- open sets in (X, T*) are β, γ ,δ, β ∨γ, β ∧γ. Now 1–β, 1–γ and 1–(β ∨γ) are fuzzy supra α- nowhere dense sets in (X, T*). But α-int*[(1–β) ∨(1–γ) ∨(1 –(β ∨γ))] = 1–(β ∧γ) ≠0. So (X,T*) is not of fuzzy supra α-Baire space.

Proposition 4.4: Let (X,T*) be a fuzzy supra topological space. Then the following are equivalent:

(1) (X,T*) is a fuzzy supra α-Baire space.
(2) α-int* (λ) = 0 for every fuzzy supra α- first category set λ in (X,T*).
(3) α-cl*(μ)=1, for every fuzzy supra α-residual set μ in (X,T*).

Proof:
(1) ⇒(2): Let λ be a fuzzy supra α-first category set in (X,T*). Then λ = ($\vee_{i=1}^{\infty}$(λ_i)), where (λ_i)'S are fuzzy supra α-nowhere dense sets in (X,T*). Now α-int*(λ)=α-int* (($\vee_{i=1}^{\infty}$(λ_i))=0. (Since (X, T*) is a fuzzy supra α- baire space). Thus α-int*(λ) =0.

(2) ⇒(3): Let μ be fuzzy supra α-residual set in (X,T*). Then 1 –μ is a fuzzy supra α-first category set in (X,T*). by hypothesis , α-int(1 –μ)=0, which implies that 1 –α-cl*(μ)=0. Thus α-cl*(μ) =1.

(3) ⇒(1): Let λ be fuzzy supra α- first category set in (X,T*) Then λ=$\vee_{i=1}^{\infty}$(λ_i) where (λ_i)' s are fuzzy supra α-nowhere dense sets in(X,T*). Now λ is a fuzzy supra α- first category set, 1 –λ is a fuzzy supra α-residual set in (X,T*). by hypothesis, we have α-cl*(1 –λ) =1, which implies that 1 –α-int*(λ)=1. Thus α-int*(λ)=0. That is , α-int*($V_{i=1}^{\infty}$ (λ_i)) = 0, where (λ_i)'s are fuzzy supra α-nowhere dense sets in(X,T*).So (X,T*) is a fuzzy supra α-Baire space.

Proposition 4.5: If the fuzzy supra toplogical space (X, T*) is a fuzzy supra α-Baire space, then (X, T*) is a fuzzy supra α-second category space.

Proof: Let (X,T*) be fuzzy supra α-Baire space . Then α-int*($V_{i=1}^{\infty}$ (λ_i)) = 0. where (λ_i)'s are fuzzy supra α-nowhere dense sets in(X,T*). Then $V_{i=1}^{\infty}$ (λ_i) $\neq 1_x$ [otherwise , $V_{i=1}^{\infty}$ (λ_i)=1_x which implies that α-int* ($V_{i=1}^{\infty}$ (λ_i)) = α-int* (1x)= 1x, which implies that 0=1,which is a contradiction]. Thus (X, T*) is a fuzzy supra α-second category space.

Remark 4.6: The converse of the above proposition need not be true. A fuzzy supra α-second category space need not be a fuzzy supra α-Baire space . For, consider the following example.

Example 4.7: Let X={a, b, c} The fuzzy supra sets β, γ and δ are defined on X as follows:

β:X → [0,1] defined as β(a)=1; β(b)=0.2; β(c)=0.7,

γ:X → [0,1] defined as γ (a)=0.3; γ (b)=0.1; γ (c)=0.2,

δ: X → [0,1] defined as δ (a)=0.7; δ (b)=0.4; δ(c)=1.

Then T= {0, β, γ, δ, β ∨ δ, β ∧ δ , 1} is clearly a fuzzy supra topology on X.

Thus the fuzzy supra α- open sets in (X,T*) are β, γ ,δ, β ∨ δ, β ∧ δ. Now 1–β, 1– δ and 1–(β ∨ δ) are fuzzy supra α-nowhere dense sets in (X, T*). Now [(1–β) ∨ (1– δ) ∨(1 –(β ∨ δ))]=1x, therefore (X,T*) is a fuzzy supra α-second category space. α-int*[(1–β) ∨(1– δ) ∨(1 –(β ∨ δ))] = 1–(β ∧ δ) ≠0. So (X,T*) is not of fuzzy supra α-Baire space.

Proposition 4.8: If the fuzzy Supra topological space (X, T*) is a fuzzy supra α-first category space, then(X, T*) is not a fuzzy supra α-Baire space.

Proof: Since (X,T*) is a fuzzy supra α- first category space, then $V_{i=1}^{\infty}$ (λ_i) = 1, where (λ_i)'s are fuzzy supra α-nowhere dense set in (X,T*). Then α-int*($V_{i=1}^{\infty}$ (λ_i)) = α-int*(1) ≠ 0, where (λi)'s are fuzzy supra α-nowhere dense sets in (X,T*). Thus (X, T*) is not a fuzzy supra α-Baire space.

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